RESEARCH ON SELF-GENERATED STOCHASTIC MOTION
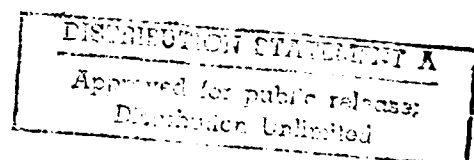
FINAL REPORT

by

A. J. Lichtenberg and M. A. Lieberman

for

Office of Naval Research

Contract N00014-84-K-0367

June 1, 1984 to September 30, 1988

ELECTRONICS RESEARCH LABORATORY

College of Engineering
University of California, Berkeley
94720

I. SUMMARY OF TECHNICAL ACCOMPLISHMENTS

The purpose of the research under this contract was to study the processes by which stochastic motion is self-generated in deterministic systems, and the consequences of the resulting stochasticity. We consider representative problems, each of which incorporates one or more of the main underlying physical phenomena. Two classes of phenomena that we have studied under this contract are diffusion across overlapping resonances, and diffusion along resonances, in more than two degrees of freedom (Arnold and modulational diffusion). The former has been studied in considerable depth, and new insights involve the fine structure of the phase space or more exact calculations. The latter phenomena still have many poorly understood aspects.

1. Hamiltonian Renormalization Theory

A systematic Hamiltonian renormalization method using regular (secular) perturbation theory has been formulated to determine at what energy the resonance structure of the system repeats itself to higher orders [1]. A self-similar property of Hamiltonian maps has been determined for p iterations of the mapping, where p is an arbitrary integer. The procedure involves the order-by-order renormalization of resonances, in which each resonance, averaged over the fast variables, locally approximates the motion of a pendulum. A standard mapping is reconstituted for each resonance. Locally, this becomes the approximate description of any mapping near a resonance. The value of the stochasticity parameter K, for self-similar behavior about a p-interaction fixed point, is then determined.

2. Corrections to Quasilinear Theory

Higher order corrections to the quasilinear diffusion coefficient can be obtained for the standard (Chirikov-Taylor) map. We have studied whether these corrections can be applied to other maps which can be locally approximated by the standard map. We have analytically obtained a local diffusion coefficient as a function of action for the Fermi map [2] which displays the characteristic oscillations in action. We have used this diffusion coefficient in the Fokker-Planck equa-

tion to obtain numerically the diffusion of an initial delta function distribution in action. We have compared this with a numerical calculation of the diffusion for the same initial distribution obtained by direct iteration of the mapping equations. The comparison demonstrates that the local approximation can be used to obtain the higher order corrections to the diffusion over much of the stochastic region for the Fermi map [2].

For some values of parameters the standard mapping exhibits accelerator modes for which the diffusion becomes infinite. For the more generic mappings the modes become quasi-modes and the diffusion is enhanced over values of the action for which the quasi-modes are present. We have studied this phenomenon and developed a procedure for calculating the diffusion in the presence of the quasi-accelerator modes by treating them as sources and sinks in the Fokker-Planck equation [3].

We have completed a study of stochasticity and resonances in the two beam accelerator [4]. The two beam accelerator is a proposed linear e^+e^- machine employing 2×500 tapered free electron lasers (FELs) as a source of microwave power. A high current, low energy (20 MeV) beam drives the FELs, producing 1 cm microwaves. These microwaves are used to accelerate a low current beam to energies of 1 TeV. High efficiencies are obtained by re-accelerating the low energy beam after passing through each FEL. Restoring the 2 MeV/particle lost on passing through each FEL section (using an induction accelerator) avoids wasting the 18 MeV/particle left in the beam. The periodic nature of the acceleration and deceleration of the low energy beam can lead to stochastic motion. We have developed design criteria that ensure that detrapping of the low energy beam by this stochasticity is minimal, preserving the high efficiencies inherent in a single tapered FEL. Numerical integrations of a one dimensional model for the FELs were performed, showing various degrees of detrapping. We also explored the effects of islands produced by resonances between the periodic acceleration and the trapped particle motion. These islands represent coherent motion of large numbers of trapped particles, leading to oscillations in the power output of the FELs.

3. Arnold and Modulational Diffusion

Our studies of Arnold diffusion in the billiards problem have been completed and a review has appeared [5]. A major advance has been the application to the ECRH problem. For one specific case of three strong resonances which are approximately equally spaced in frequency, the thin layer Arnold diffusion along a resonance layer has been treated theoretically, yielding diffusion coefficients in reasonable agreement with numerically determined values [6]. This mechanism, which can cause particles to diffusively heat across an adiabatic barrier, is found to be somewhat weaker than collisional diffusion for typical plasma confinement devices. However, Arnold diffusion could prove significant for additional heating of plasmas having higher temperature (or lower density) [7].

In modulational diffusion an oscillation in one degree of freedom is modulated at a slow driving frequency, yielding a broad band of stochasticity. The coupling of this stochastic motion to other degrees of freedom is known as modulational diffusion. The diffusion coefficient has been evaluated analytically, without the need for adjustable fitting parameters. The coefficient has been compared with numerical calculations over a wide range of system parameters and for diffusion rates varying over many orders of magnitude. The numerical and analytical results are in good agreement [8].

4. Intrinsically Stochastic Heating Mechanisms

A study of two frequency heating in the Fermi map showed that using two frequencies gave an approximately twofold increase in energy for the position of the lowest KAM barrier to stochastic heating when the resonances associated with the two frequencies were properly interlaced. We have extended this work to consider the more practical problem of two-frequency electron cyclotron resonance heating (ECRH) in mirror-confined plasmas. The ECRH is modeled by means of a four dimensional simplectic mapping derived from the nonrelativistic single particle energy change due to the spatially separate resonance zones. Fixed points have been located and

their linear stability determined analytically. Resonances in action space have been calculated and used to obtain the adiabatic barrier. Analytical expressions were derived for the quasilinear diffusion coefficients in the stochastic regime below the adiabatic barrier and found to agree well with numerical calculations. Intrinsic diffusion in the parallel energy is shown to lead to axial losses that can rival those induced by collisions alone. Detailed papers describing these results have been published [6,7]. A new phase space reconnection mechanism was uncovered and explored in a simpler system [9].

II. PUBLISHED PAPERS

[1] Lichtenberg, A. J., "Universality from Resonance Renormalization of Hamiltonian Maps," *Physica 14 D*, 387 (1985).

[2] Murray, N., M. A. Lieberman, and A. J. Lichtenberg, *Phys. Rev. A 32*, 4 (1985).

[3] Lichtenberg, A. J., M. A. Lieberman, and N. W. Murray, "The Effect of Quasi-Accelerator or Modes on Diffusion," *Physica 28D*, 371 (1987).

[4] Murray, N. W. and M. A. Lieberman, "Stochasticity and Resonance in the Two Beam Accelerator," Proceedings of the Symposium on Advanced Accelerator Concepts, Am. Inst. of Physics (1988).

[5] Lieberman, M. A. "Many Dimensional Hamiltonian Systems," *Physics of High Energy Particle Accelerators*, A.I.P. Proceedings, 153, 631 (1987).

[6] Howard, J. E., A. J. Lichtenberg, M. A. Lieberman, and R. H. Cohen, "Four Dimensional Mapping Model for Two Frequency ECH," *Physica 20D*, 259 (1986).

[7] Lichtenberg, A. J., J. E. Howard, M. A. Lieberman, and R. H. Cohen, "Velocity Diffusion in Two-Frequency Electron Cyclotron Resonance Heating," *Physics of Fluids 29*, 1061 (1986).

[8] Chirikov, B. V., M. A. Lieberman, D. L. Shepelyansky, and F. M. Vivaldi, "A Theory of Modulational Diffusion," *Physica 14D*, 289 (1985).

[9] Howard, J. E. and S. M. Hohs, "Stochasticity and Reconnection in Hamiltonian Systems," *Phys. Rev. A*, 29, 418 (1984).